$$\tilde{M}_{\varphi\varphi}{}^{(1,2)} = -(\tilde{S}/\tilde{R}_2, \bar{\sigma}) + (1, \epsilon)\cdot$$
$$[(d\tilde{V}^{(1,2)}/d\bar{s}) + \nu\tilde{V}^{(1,2)}\cot\varphi/\tilde{R}_2 + (1-\lambda)\cdot\bar{\epsilon}_\theta{}^{(1,2)}/\tilde{R}_2]/(1+\nu)$$
$$\tilde{M}_{\theta\theta}{}^{(1,2)} = -(\lambda\tilde{S}/\tilde{R}_2, \bar{\sigma}) + (1, \epsilon)\cdot$$
$$[\nu\cdot(d\tilde{V}^{(1,2)}/d\bar{s}) + \tilde{V}^{(1,2)}\cot\varphi/\tilde{R}_2 - (1-\lambda)\cdot\bar{\epsilon}_\theta{}^{(1,2)}/\tilde{R}_2]/(1+\nu)$$

Implicit in the equations of equilibrium is the restriction that the shell is otherwise unloaded.

Approximate Solution for Middle Surface Heating

It follows from the preceding equations that all barred quantities are of order unity, save for the middle surface stresses that are of order (ϵ). Thus, it follows that a satisfactory solution is obtained by writing

$$\bar{\epsilon}_\varphi{}^{(1)}, \bar{\epsilon}_\theta{}^{(1)} = \tilde{S} \qquad \tilde{V}^{(1)} = -\tilde{R}_2\cdot\frac{d\tilde{S}}{d\bar{s}}$$

$$(1+\nu)\cdot\tilde{M}_{\varphi\varphi}{}^{(1)} = (d\tilde{V}^{(1)}/d\bar{s}) + \nu\tilde{V}^{(1)}\cot\varphi/\tilde{R}_2 - (\lambda+\nu)\tilde{S}/\tilde{R}_2$$
$$(1+\nu)\cdot\tilde{M}_{\theta\theta}{}^{(1)} = \tilde{V}\cot\varphi/\tilde{R}_2 + \nu\cdot(d\tilde{V}/d\bar{s}) - (1+\lambda\nu)\tilde{S}/\tilde{R}^2$$

However, for $\tilde{S}$ = const = 1, say, we find $\tilde{V}^{(1)} = 0\ (\epsilon)$, so that the approximate solution becomes

$$\bar{\epsilon}_\varphi{}^{(1)}, \bar{\epsilon}_\theta{}^{(1)} = 1 \qquad (1+\nu)\cdot\tilde{M}_{\phi\phi}{}^{(1)} = -(\lambda+\nu)/\tilde{R}_2$$
$$(1+\nu)\tilde{Q}_\varphi{}^{(1)} = -\ [\tilde{R}_2\cdot(d\lambda/d\bar{s}) - (1-\lambda^2)\cdot\cot\varphi]/\tilde{R}_2{}^2$$

The behavior of the solution in the neighborhood of the apex can be observed in the limiting case of a sphere of radius a by taking $\tilde{S} = 1, R_c = a$, i. e.,

$$\tilde{N}_{\phi\phi}{}^{(1)}, \tilde{N}_{\theta\theta}{}^{(1)}, \tilde{V}^{(1)}, \tilde{Q}_\varphi{}^{(1)} = 0$$
$$\tilde{M}_{\phi\phi}{}^{(1)}, \tilde{M}_{\theta\theta}{}^{(1)} = -1 \qquad \bar{\epsilon}_\varphi{}^{(1)}, \bar{\epsilon}_\theta{}^{(1)} = 1$$

Approximate Solution for a Temperature Gradient

For this case, all barred quantities are of order of magnitude unity, and hence a satisfactory solution is obtained by taking

$$\tilde{M}_{\varphi\varphi}{}^{(2)}, \tilde{M}_{\theta\theta}{}^{(2)} = -\bar{\sigma} \qquad \tilde{Q}_\varphi{}^{(2)} = -\frac{d\bar{\sigma}}{d\bar{s}}$$

$$\tilde{N}_{\theta\theta}{}^{(2)} = -\frac{d}{d\bar{s}}\left(\tilde{R}_2\cdot\frac{d\bar{\sigma}}{d\bar{s}}\right) \qquad \tilde{N}_{\phi\phi}{}^{(2)} = -\frac{d\bar{\sigma}}{d\bar{s}}\cdot\cot\varphi$$

$$\bar{\epsilon}_\varphi{}^{(2)} = \bar{\sigma}(1-\lambda\nu)/(1-\nu)\tilde{R}_2 + (\tilde{N}_{\phi\phi}{}^{(2)} - \nu\tilde{N}_{\theta\theta}{}^{(2)})/(1-\nu)$$
$$\bar{\epsilon}_\theta{}^{(2)} = \bar{\sigma}\cdot(\lambda-\nu)/(1-\nu)\tilde{R}_2 + (\tilde{N}_{\theta\theta}{}^{(2)} - \nu\tilde{N}_{\phi\phi}{}^{(2)})/(1-\nu)$$
$$\tilde{V}^{(2)} = -\tilde{R}_2\cdot(d\bar{\epsilon}_\theta{}^{(2)}/d\bar{s}) +$$
$$(1+\nu)\ [\bar{\sigma}(1-\lambda)/\tilde{R}_2 + \tilde{N}_{\phi\phi}{}^{(2)} - \tilde{N}_{\theta\theta}{}^{(2)}]\cdot\cot\phi/(1-\nu)$$

However, for $\bar{\sigma}$ = const = 1, say, we find

$$\tilde{Q}_\varphi{}^{(2)}, \tilde{N}_{\varphi\varphi}{}^{(2)}, \tilde{N}_{\theta\theta}{}^{(2)} = 0(\epsilon)$$

so that the approximate solution becomes

$$\bar{\epsilon}_\varphi{}^{(2)} = (1-\lambda\nu)/\tilde{R}_2(1-\nu) \qquad \bar{\epsilon}_\theta{}^{(2)} = (\lambda-\nu)/\tilde{R}_2(1-\nu)$$
$$(1-\nu)\tilde{V}^{(2)} = -\tilde{R}_2\ (d/d\bar{s})\ [(\lambda-\nu)/\tilde{R}_2] +$$
$$(1+\nu)(1-\lambda)\cot\varphi/\tilde{R}_2$$

The behavior of the solution in the neighborhood of the apex can be observed in the limiting case of a sphere of radius a by taking $\bar{\sigma} = 1, R_c = a$, i. e.,

$$\tilde{N}_{\phi\phi}{}^{(2)}, \tilde{N}_{\theta\theta}{}^{(2)}, \tilde{V}^{(2)}, \tilde{Q}_\varphi{}^{(2)} = 0$$
$$\tilde{M}_{\varphi\varphi}{}^{(2)}, \tilde{M}_{\theta\theta}{}^{(2)}, \bar{\epsilon}_\varphi{}^{(2)}, \bar{\epsilon}_\theta{}^{(2)} = 1$$

Concluding Remarks

As mentioned previously, the complete solution that satisfies a set of boundary conditions is obtained by superposing on the preceding particular solution a solution for arbitrary edge loading. If the shell is shallow, the results given in Ref. 2 are applicable in general, and Ref. 3 may be applied to spherical shells in particular. Alternatively, if the shell is not shallow, the method of asymptotic integration as proposed in Ref. 4 is applicable. The accuracy of the resulting composite solution depends, of course, on the region of the shell and on the accuracy of the edge-loading solution.

Finally, it can be shown that the preceding technique for constructing a particular solution is applicable to nonaxisymmetric temperature distributions provided that the circumferential variation may also be considered as slowly varying. For this more general problem, the boundary conditions may be satisfied by superposing a solution to the shallow shell equations, if applicable, or the solution proposed in Ref. 5.

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Zero Angle-of-Attack Sensor

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It is desirable to have under certain conditions a re-entry vehicle enter at zero angle of attack and maintain this angle during re-entry. In order to control flight at zero angle of attack, it is necessary to sense deviations from zero. One method for detecting angle of attack is shown in Fig. 1. Two σU transducers[1,2] are mounted on either face of a wedge. The signal from transducer 1 is less than the signal from transducer 2. A control system activates flaps or reaction jets to equalize the signals from the two transducers.

The σU transducers, which are described in Refs. 1 and 2, generate a voltage proportional to the product of electrical conductivity and local flow velocity. The relation between signal voltage e and σU is[1]

$$e = \int_{\text{ablation surface}}^{\text{shock wave}} \sigma U K dy \tag{1}$$

where K is the influence function for the transducer, and y is distance normal to the flow. Flight transducers with K (at $y = 0$) = 10 mv/(mho/m)(m/sec)(m) can be readily built.

To determine the change in signal for a change in angle of attack, σU for flow over a wedge was calculated. A free-stream velocity of V_∞ = 26,000 fps was chosen. Using Feldman's[3] tables, ρ_w, T_w, and V_w were obtained; the notation of the tables is used here. The velocity along the wedge V_w is equal to U.

Received April 14, 1964.
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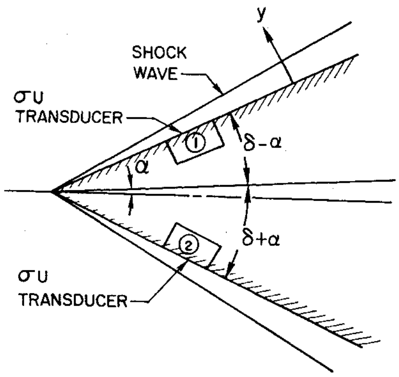

Fig. 1 The signal of transducer 2 is larger than that of transducer 1. The difference in signals can be used by the control system to properly activate the flaps or reaction jets.

Electrical conductivity σ was found from graphs appearing in the paper by Viegas and Peng.[4] The results are shown in Fig. 2 for five different altitudes. Each curve is shifted 2.5° to prevent overlapping the curves. On the windward side, increased deflection angle increases T and ρ, which, in turn, increase σ. However, the increased deflection angle decreases the flow velocity behind the shock wave. Increase in σ is relatively larger than the decrease in U, so that σU does increase as shown in Fig. 2.

Consider a wedge with half angle of 23.5° flying at zero angle of attack at 100,000 ft. The value of σU at the transducer is 3.6×10^3 mho/sec. Now assume an angle of attack of 1°. The transducer on the "windward" side (number 2 in Fig. 1) is exposed to a σU of 4.8×10^3, and the transducer on the "leeward" side sees σU of 2.3×10^3. The derivative of the fractional signal change is

$$\frac{\partial(\Delta e/e)}{\partial\alpha} = \frac{e_2 - e_1}{e\alpha} = \frac{4.8 - 2.3}{(3.6)(1°)} = \frac{0.69}{\text{deg}} \tag{2}$$

There is a large change in signal for a change in angle of attack. Changes in signal level with changes in angle of attack have been observed in flight (see Fig. 10 of Ref. 2).

It is of interest to compare the fractional pressure change for the same conditions. For Newtonian flow, the pressure varies as the sine of the deflection angle squared. Hence, for the example,

$$\frac{\partial(\Delta p/p)}{\partial\alpha} = \frac{\sin^2 24.5° - \sin^2 22.5°}{\sin^2 23.5°} = \frac{0.16}{\text{deg}} \tag{3}$$

For the example considered, the pair of σU transducers gives a larger fractional signal change than a pair of static pressure taps.

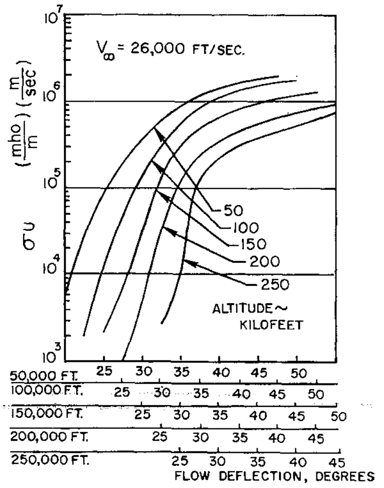

Fig. 2 Electrical conductivity/velocity for flow over a wedge as a function of deflection angle.

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A Family of Similar Solutions for Axisymmetric Incompressible Wakes

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Introduction

IN Ref. 1, Stewartson found a family of solutions of the Falkner-Skan equation

$$f''' + ff'' + \beta(1 - f'^2) = 0$$

for "free streamline" flows, i.e., with boundary conditions

$$(0) = f''(0) = 0; \quad \lim_{\eta\to\infty} f'(\eta) = 1$$

He showed that for positive pressure gradients ($-\frac{1}{2} < \beta < 0$) the solutions exhibit a wake-like character ($f'(0) < 1$) and that for $-0.1988 < \beta < 0, f'(0) < 0$. Recently it has been recognized[2] that these solutions, or actually their compressible analogs,[3] possess several important features displayed by flows in the base flow region or near wake of high speed objects.

In the present note, a new family of similar solutions is derived for axisymmetric, incompressible wakes. It is shown that, just as in the case of the two-dimensional Stewartson family, these solutions contain the Chapman[4] constant pressure mixing solution and the solution for uniform flow in the axial component of velocity as limiting cases.

Equations and Boundary Conditions

The continuity and momentum equations for large Reynolds number, incompressible axisymmetric flow are the following[5]:

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial(vr)}{\partial r} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = u_e\frac{du_e}{dx} + \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{2}$$

Let

$$\psi = c_1 r x^m f(\eta) \qquad \eta = c_2 r/x^n \tag{3}$$

Received April 14, 1964.

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